the divergence between coefficients α_g obtained with high inertia sensors and an inertia calculated by Eq. (1) gradually decreases.

The character of the change in heat liberation coefficient exerted no marked effect on sensor inertia (signals of various form were considered: N- and П-shaped, trapezoidal and intermediate forms).

Mathematical modeling results indicate that if the deviation of instantaneous α_g values from the mean value comprises less than 15-20% the gain in accuracy achieved by use of sensors with the required inertia is not great.

Increase in the frequency of α_g change imposes more rigid limitations on the sensor inertia.

NOTATION

T, wall temperature; T_g, hot gas temperature; T_a, air temperature; T_i, initial wall temperature; α_g, heat liberation coefficient between hot gas and wall; α_a, heat liberation coefficient between air and wall; a, thermal diffusivity coefficient of wall material; λ, thermal conductivity of wall material; δ, wall thickness; A and B, steady state temperature field coefficients; x, coordinate; τ, time.

EFFECT OF TEMPERATURE MEASUREMENT ERRORS ON THE ACCURACY OF BOUNDARY CONDITIONS

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Results are presented from an evaluation of the accuracy of the solution of an inverse heat-conduction problem for an infinite plate with nonsymmetrical heat-transfer boundary conditions.

The accuracy of the determination of boundary conditions for heat transfer was analyzed in relation to the accuracy of the input data (temperature measurements), with application to the measurement of nonsteady heat flows by means of alpha calorimeters. Assuming that the temperature field within the sensitive element of the calorimeter is kept uniform, it is possible to reliably determine the coefficient of heat transfer between the end of the calorimeter core and the flow of heat carrier by using the solutions of the inverse heat-conduction problem for an infinite plate. Then the temperature field of the sensitive element of the gradient alpha calorimeter is described by the Fourier equation

$$\frac{\partial\Theta(X,\ \mathrm{Fo})}{\partial\,\mathrm{Fo}} = \frac{\partial^2\Theta(X,\ \mathrm{Fo})}{\partial X^2} \tag{1}$$

with the boundary conditions

$$\frac{\partial\Theta(1,\ \mathrm{Fo})}{\partial X} = \mathrm{Bi}(\mathrm{Fo})\,[\Theta_c(\mathrm{Fo}) - \Theta(1,\ \mathrm{Fo})], \tag{2}$$

$$\Theta(0,\ \mathrm{Fo}) = \Theta_0,\ \Theta(X,\ 0) = f(X). \tag{3}$$

Given experimental values of the temperature of the heat carrier $\Theta_c(\mathrm{Fo})$ and the temperature on the heated end of the sensitive element $\Theta(1,\ \mathrm{Fo}) = \varphi(\mathrm{Fo})$, the solution of the problem for these boundary conditions has the form [1]

$$\mathrm{Bi}(\mathrm{Fo}) = \{[\varphi(\mathrm{Fo}) - \varphi(0)] + 2\sum_{n=1}^{\infty}[\varphi(\mathrm{Fo}) - Y_n(\mathrm{Fo})]\}/[\Theta_c(\mathrm{Fo}) - \varphi(\mathrm{Fo})], \tag{4}$$

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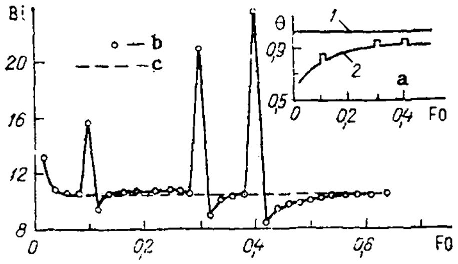

Fig. 1

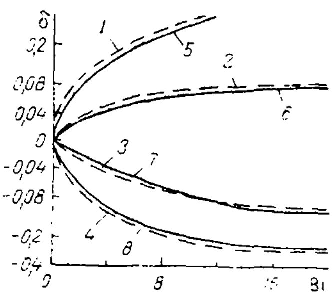

Fig. 2

Fig. 1. Results of solution of the inverse heat-conduction problem: a) initial data [1) temperature of the heat carrier; 2) temperature of the end of the rod at $Bi_0 = 10$]; b) solution in the presence of random errors in the measurement of the temperature of the end of the rod; c) solution when the temperatures are entered without distortions.

Fig. 2. Effect of errors of measurement of the temperature of the heat carrier $\Theta_c(Fo)$ and the end of the rod $\varphi(Fo)$ on accuracy of determination of the heat-transfer boundary conditions: 1, 2, 3, 4) $\Delta\Theta_c = -0.03$; -0.01; 0.01; 0.03 at $\Delta\varphi = 0$; 5, 6, 7, 8) $\Delta\varphi = 0.03$; 0.01; -0.01; -0.03 at $\Delta\Theta_c = 0$.

where $Y_n(fo)$ is the solution of the system of equations

$$\frac{1}{\mu_n^2}\dot{Y}_n(\mathrm{Fo}) + Y_n(\mathrm{Fo}) = \varphi(\mathrm{Fo}),\ n = 1,\ 2,\ \ldots;\ \mu_n = n\pi. \tag{5}$$

Solution (4) was obtained using the method proposed in [2] for solving an inverse heat-conduction problem.

We propose to solve system (5) by the finite differences method. This approach is valid because experimental data on temperature is usually obtained in the form of tables of values over certain intervls of time ΔFo. After $\dot{Y}_n(Fo)$ is replaced by finite differences, the recursion formula for the determination of $Y_n(Fo)$ has the form

$$Y_{n,k} = [Y_{n,k-1} + \mu_n^2 \Delta\,\mathrm{Fo}\,\varphi(k\Delta\,\mathrm{Fo})]/(1 + \mu_n^2 \Delta\,\mathrm{Fo}). \tag{6}$$

With allowance for the finite number of terms of the series, Eq. (4) is changed to the form

$$\mathrm{Bi}(k\Delta\,\mathrm{Fo}) = \frac{[\varphi(k\Delta\,\mathrm{Fo}) - \varphi(0)] + 2\sum_{k=1}^{N}[\varphi(k\Delta\,\mathrm{Fo}) - Y_{n,k}]}{\Theta_c(k\Delta\,\mathrm{Fo}) - \varphi(k\Delta\,\mathrm{Fo})}. \tag{7}$$

The number of terms in the series can be determined on the basis of the condition of ensuring a prescribed degree of approximation of the original function by $Y_{n,k}$, i.e., $|\varphi(k\Delta Fo) - Y_{n,k}| < \varepsilon$.

To realize a solution, we wrote a program for an ES series computer. The initial data for the solution of the control problems was the temperature distribution in the plate [3]. The calculations were performed for the range of Biot numbers $0.5 \le Bi_0 \le 10$. The time step was varied within the range $0.01 \le \Delta Fo \le 0.1$.

Analysis of the results shows that at values of the time interval $0.02 \le \Delta Fo \le 0.04$, the error of the solution of the inverse problem $\delta_e = (Bi_1 - Bi_0)/Bi_0$ is no greater than 3%. Here, Bi_1 is the value of the Biot criterion obtained from the solution of (7) and averaged over the time interval $N\Delta Fo$.

The effect of the accuracy of the initial data was analyzed by the sample error method. The size of the systematic and random errors in ambient temperature and the temperature of the end of the calorimeter core is determined by the degree of approximation of the solution of the direct problem.

It can be seen from Fig. 1 that the influence of random errors in the data on the temperature of the end ($\Delta\varphi = 0.1\varphi(\mathrm{Fo})$) does not extend to the previous moments of time and relaxes over the next 2-3 solution steps. Possible random errors in the measurement of the heat carrier temperature also lead only to local perturbations and do not disturb the convergence of the solution.

The systematic error in the temperature values is given as follows:

for the temperature of the end of the sensitive element

$$\varphi'(\mathrm{Fo}) = \varphi(\mathrm{Fo}) \pm k\varphi(\mathrm{Fo}),$$

for the temperature of the heat carrier

$$\Theta'_c(\mathrm{Fo}) = \Theta_c(\mathrm{Fo}) \pm k\Theta_c(\mathrm{Fo}),$$

where $\varphi'(\mathrm{Fo})$ and $\Theta_c'(\mathrm{Fo})$ are approximate values of temperature; $\varphi(\mathrm{Fo})$, $\Theta_c(\mathrm{Fo})$ are exact values of temperature; k successively takes values of 0.01, 0.03, 0.05, and 0.1.

Figure 2 shows graphs for determining the corresponding errors in the calculated rate of heat transfer $\delta_\varphi = (\mathrm{Bi} - \mathrm{Bi}_0)/\mathrm{Bi}_0 = f(\mathrm{Bi}_0, \Delta\varphi)$ and $\delta_c = (\mathrm{Bi} - \mathrm{Bi}_0)/\mathrm{Bi}_0 = f(\mathrm{Bi}_0, \Delta\Theta_c)$.

The total error of the heat-transfer boundary conditions is determined not only by the absolute values of its components δ_φ and δ_c, but also by their signs. When the systematic errors in the measurement of the temperatures φ and Θ_c are of the same sign, the calculated values of the Biot criterion are nearly the same as the result of the solution of the inverse problem in the case where the initial data is entered without distortions. The presence of errors of different signs leads to serious loss of accuracy. In this case, the total error of the solution can be found by adding the absolute values of δ_φ and δ_c — each of which is determined from Fig. 2. The following relation can be used to evaluate the total relative error

$$\delta_0 = (1 + \delta_e)(1 + \delta_\Sigma) - 1.$$

The data in Fig. 2 can be used to evaluate the allowable values of error in the temperature measurement, as well as to choose design parameters for alpha calorimeters. Thus, for example, an accuracy $\delta_0 \leq 20\%$ is assured in the solution of the inverse problem within a broad range of Biot numbers when the systematic error of the temperature measurement is no greater than 1%. It can be seen from Fig. 2 that the error of the solution decreases with a decrease in the Biot number. In connection with this, the sensitive element of the calorimeter should be as small as possible. It is best to use materials with a high thermal conductivity to make the element. Given a suitable choice of material (λ) and dimensions (R) for the core of the calorimeter, it is possible to solve the inverse problem with a maximum error of 5-7%.

NOTATION

$\Theta(X, \mathrm{Fo})$, relative temperature; X, relative coordinate; $\mathrm{Fo} = a\tau/R^2$, dimensionless time; $\Delta\mathrm{Fo}$, step of Fourier number; $\mathrm{Bi} = \alpha R/\lambda$, Biot criterion; $\varphi(\mathrm{Fo}) = \Theta(1, \mathrm{Fo})$, temperature of heated end of rod; Θ_c, temperature of heat carrier; $\Delta\varphi$, error of wall-temperature measurement; $\Delta\Theta_c$, error of measurement of heat-carrier temperature; δ_φ, error of determination of Biot number in the presence only of a wall-temperature measurement error; δ_c, error of determination of heat-transfer rate in the presence only of an error in measurement of heat-carrier temperature; δ_Σ, total error of determination of Biot number from inaccuracies in temperature measurement; δ_0, total relative error.

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